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A comparison of the uniaxial tensile and pure bending strength of SiC filaments

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The uniaxial and bend strengths of AVCO-SCS-6 SiC filaments were experimentally determined to be 3250 and 4320 MPa, respectively, at room temperature. Using Weibull statistics, a relationship between the uniaxial and bend strength of the fibres was derived and was found to compare favourably with experimentally determined strength values. The elastic modulus of the filaments was also determined, using a gauge length extrapolation technique.

1. Introduction

SiC filaments are under investigation for use in a variety of metal and ceramic matrix composites because of their excellent strength, stiffness and thermal stability [1, 2]. Because the composite strength and elastic modulus are directly related to the filament modulus, strength, and strength distribution, it is important to fully characterize the filament. Although filament strength is usually and adequately determined by uniaxial tensile tests, there are certain disadvantages to this technique. Although the tensile test method described subsequently is reasonably rapid to conduct, some time must be expended for each filament. Moreover, a well-aligned tensile machine is required. It would be desirable if a rapid and simpler test were available to measure the failure strength of a large number of filaments. Such a technique is particularly desirable for examining the properties of layers of filaments removed (by etching) from a composite tensile specimen for signs of damage. In this note, such a method involving pure bending is described. The strength values determined by this method were then related to the uniaxial tensile strength using Weibull statistics.

2. Experimental procedure

2.1. Material

A spool of continuous Avco SCS-6 SiC filaments (Avco Corp., Lowell, Massachusetts) was used for these studies. This type of filament has a 30 μm diameter carbon core which is coated by chemical vapour deposition with about 55 μm of SiC. The SiC surface has two 1.0 to 1.5 μm thick pyrolytic carbon layers and the total filament diameter is about 142 μm. A photomicrograph of such a filament is shown in Fig. 1. The carbon core serves as a substrate for the deposition of the SiC, which is the structural part of the filament; the carbon surface layers are intended to minimize interaction between the SiC and the composite matrix. As part of their quality control the manufacturers had measured the strength of the filaments on the spool as 3150 MPa. The strength of the filaments used in this study was somewhat below the usual 3450 to 4140 MPa strength generally credited to this type of filament. The manufacturers give the value of the modulus of SCS-6 filaments to be 400 GPa.

2.2. Uniaxial elastic modulus and strength measurements

The elastic modulus was measured by a technique utilizing the inherent spring stiffness of an Instron tensile testing machine [3]. In this method, the total crosshead motion, δ_T, of a filamentary specimen held between two grips in a tensile machine is given by the specimen deflection, δ_s, and the machine contribution, δ_m. They can be expressed as

$$\delta_s = Pl/AE \quad (1)$$

$$\delta_m = KP \quad (2)$$

where P is load, l the specimen length, A the specimen area and E the elastic modulus. K is taken to represent the machine stiffness. The total deflection is the sum of δ_s and δ_m, or

$$\delta_T = \frac{Pl}{AE} + KP \quad (4)$$

The apparent elastic modulus, E^*, is given by the slope of the experimentally determined load–deflection curve and includes the deflection contributed by the testing machine. It is given by

$$E^* = Pl/A\delta_T \quad (4)$$

Substituting the value of δ_T from Equation 3 into Equation 4 and slightly rearranging,

$$\frac{1}{E^*} = \frac{AK}{l} + \frac{1}{E} \quad (5)$$

A plot of the reciprocal of the apparent elastic modulus, E^*, $1/l$ will yield a straight line that extrapolates to the true filament elastic modulus E at $l = \infty$ ($1/l = 0$).

The strengths of the SiC filaments in uniaxial tension were measured in an Instron testing machine using air-actuated flat-faced grips lined with aluminium foil. The gauge length of the filament was

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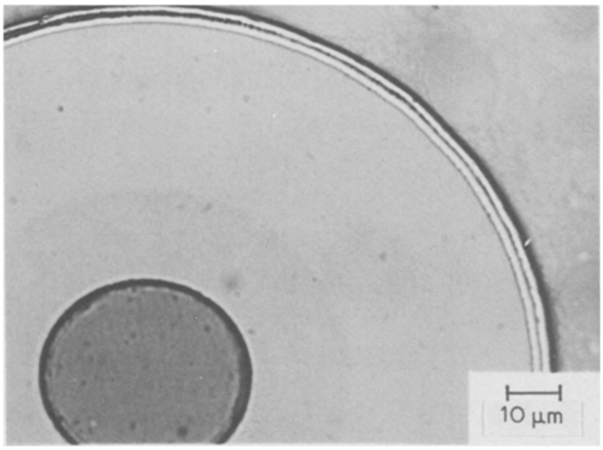

Figure 1 Cross-sectional micrograph of AVCO SCS-6 SiC filament.

12.7 cm and the crosshead rate 0.13 cm min^{-1}. This procedure has been previously described [3]. The filaments were surrounded by tissue paper during the test to capture the filament pieces after fracture; because of their high strength the fibres failed explosively. Because of the shock waves travelling the length of the filament after failure, it was not possible to assign a failure point in the fibre length.

2.3. Filament bending strength

Bending strains were imposed on SiC filaments by forcing the filaments to conform to a known radius of curvature. This was accomplished by looping a filament around a series of successively smaller drill bits until the filament failed. The drill bit diameters bracketed the strain-to-fail of the filament. The outer fibre strain can be estimated from the equation

$$\varepsilon = d/D \qquad (6)$$

where d is the filament diameter and D the drill bit diameter. Because the entire fibre length looped about the drill bit is subjected to the same strain, the situation corresponds to that of a beam under a pure bending load. The bending strength for such a filament can be calculated from

$$\sigma = \varepsilon E \qquad (7)$$

where ε is the strain obtained from Equation 6 above, E the elastic modulus and σ the filament strength. The filament failure strain was taken as the average of the strain imposed by the drill bit which caused failure and the preceding one which did not.

To measure the strain-to-fail and the strength of a large number of filaments, and thus obtain statistical data, a variation of the above procedure was used. About 20 to 30 filaments having a length of four inches (102 mm) or more were laid on a flat surface and arranged so that they were all parallel and touching along their lengths. A piece of masking tape was placed transversely over the first half-inch (13 mm) of the filament array. Three inches (76 mm) away from the edge of the first piece of tape a second piece of masking tape was attached. The two pieces of masking tape were lifted along with the filament array off the flat surface and folded over on to the opposite side of the filament array to securely hold them. Using such an arrangement, the entire filament array could be bent around a series of successively smaller drill bits. For each drill size the number of filaments which failed could be used to calculate the distribution of filament bend strengths. When a filament broke, it would be broken free from the tapes holding it such that it would not interfere with further measurements. The distance between the two pieces of tape defined the gauge length for the experiment. Any gauge length could be potentially used as long as every portion of the filament array was forced to conform to the drill bit radius of curvature. Failures at the edge of the tape were discounted. After a learning period, 20 to 30 filaments could be tested in about 30 min.

3. Results and discussion

3.1. Uniaxial modulus and strength measurements

The modulus of the SiC filament was determined in the manner previously described. The apparent elastic modulus was measured from the experimentally obtained load–deflection curve of the filamentary specimen, at 10 separate gauge lengths ranging from 25.4 to 2.54 cm. At least three separate readings were obtained at each gauge length (the test was stopped prior to failure). As in the case of the tensile tests, the specimens were gripped in air-actuated flat-faced grips lined with aluminium foil.

The results are shown in Fig. 2, where the reciprocal of the apparent elastic modulus obtained from the load–deflection curves is shown as a function of the reciprocal gauge length. A least-squares line fitted to the data intercepts the ordinate at a value of 2.51×10^{-3} GPa^{-1}, yielding an SiC modulus value of 398 GPa. This is in excellent agreement with the manufacturer's value of 400 GPa.

The strength of the SiC fibres, tested as previously described, is shown in Table I and plotted in Fig. 3. The gauge length for all these tests was 12.7 cm and the deflection rate 1.27 mm min^{-1}. The values in Table I have been ordered to facilitate the graphical determination of the Weibull modulus, m. As described in the following section, a simple method of determining the Weibull modulus involves plotting the natural

TABLE I Ordered tensile strength of SiC filaments*

Ordering number, N	Probability of fracture, $N/(N-1)$	Failure stress† (MPa)
1	0.0667	2940
2	0.133	3020
3	0.200	3140
4	0.267	3180
5	0.333	3190
6	0.400	3260
7	0.467	3260
8	0.533	3360
9	0.600	3360
10	0.667	3360
11	0.733	3360
12	0.800	3360
13	0.867	3380
14	0.933	3430

*Gauge Length = 12.7 cm; crosshead deflection rate 1.27 mm min^{-1}.
†Average tensile strength = 3260 ± 146 MPa.

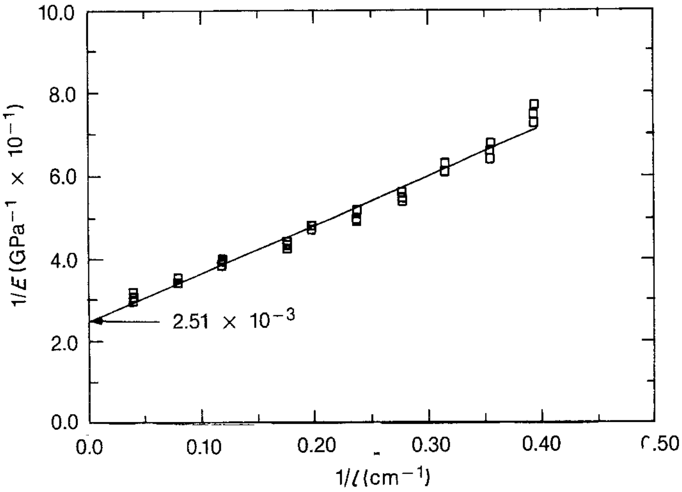

Figure 2 Plot of reciprocal apparent modulus against reciprocal gauge length for fibre modulus determination.

logarithm of the ordered strength or the abscissa against the double natural logarithm of the reciprocal of $(1 - S)$, where S is the failure probability. When the data in Table I are plotted in this manner, the least-squares slope is the value of the Weibull modulus, m; in this case, $m = 22.3$. This is an unusually high value for m, which generally lies in the range of about 5 to 15 for most structural ceramics [4].

3.2. Flexural strength

Twenty-five SiC filaments were aligned and taped to form a three-inch (76 mm) gauge length as described above. Table II shows the results of the bending experiments. The data indicate that the SiC filaments have a narrow strength distribution with an average value of 4320 MPa and a standard deviation about the mean of 192 MPa. During the bending experiments the carbon-rich SiC surface coatings were observed to spall from the filaments, several drill bit diameters prior to the failure of individual filaments. This suggests that the carbon-rich surface layer is not well bonded to the SiC and that the surface layer has a lower strain-to-fail in bending than does the SiC. In addition, the measured filament diameter of 142 μm was somewhat high due to the loss of the surface layer. This represents a systematically 2% higher estimate of the strain-to-failure values.

3.3. Relation between uniaxial and flexural strength

Weibull statistics are widely used to relate the strengths of brittle materials obtained under different conditions of loading [5, 6]. The strength variation of such brittle materials can be characterized by a weakest-link model given by

$$P = 1 - e^{-R} \tag{8}$$

where P is the probability of failure and R the risk of rupture. For a two-parameter (σ_0, m) distribution the risk of rupture is defined as the integral of the stress, σ, acting on the volume under stress, V. The expression is

$$R = \int_V \left(\frac{\sigma}{\sigma_0}\right)^m \mathrm{d}V \tag{9}$$

It is more usual to express the risk of rupture in terms of a load factor, k, as

$$R = kV\left(\frac{\sigma_{\max}}{\sigma_0}\right)^m \tag{10}$$

Here σ_0 is a normalizing constant, m the Weibull modulus and $\sigma_{\max}$ the maximum stress.

The Weibull modulus m, which is a measure of the scatter of the data, can be estimated in the simplest manner by a rearrangement of Equations 8 and 9:

$$\ln \ln \left(\frac{1}{1 - P}\right) = m \ln \sigma + \text{constant} \tag{11}$$

The probability P is calculated from $n/(N + 1)$, where n is the ordering number of the ordered strengths σ and N the total number of specimens. The slope of Equation 11 gives the Weibull modulus.

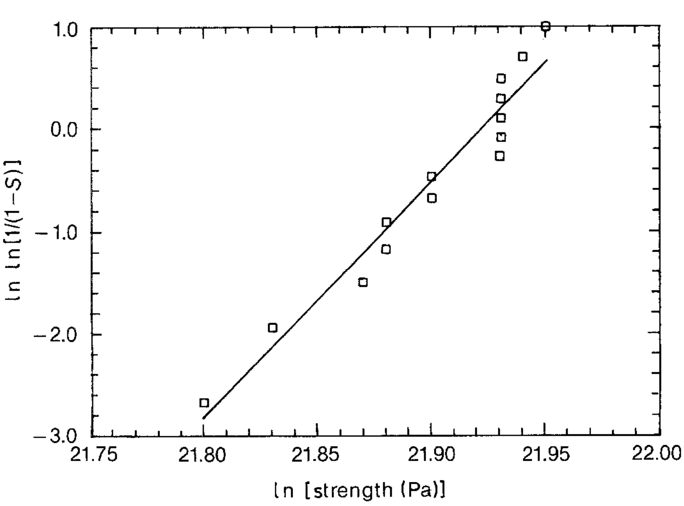

Figure 3 Plot probability of failure against filament strength for Weibull modulus determination. Weibull modulus = 22.3.

TABLE II Pure bending strength of SiC filaments*

Drill diameter		Filament strain	Filament stress (MPa)	Average failure stress (MPa)†	Failures
(cm)	(in.)				
2.550	(1.000)	0.00560	2230		0
2.223	(0.875)	0.00640	2550	2390	0
1.905	(0.750)	0.00747	2980	2760	0
1.588	(0.625)	0.00896	3580	3280	0
1.509	(0.594)	0.00943	3760	3670	1
1.430	(0.563)	0.00995	3970	3860	1
1.349	(0.531)	0.01055	4210	4090	1
1.270	(0.500)	0.01120	4470	4340	18
1.229	(0.484)	0.01157	4620	4540	4

*Assumed modulus = 399 GPa; filament diameter = 0.142 mm; gauge length = 7.62 cm.
†Average bending strength = 4320 ± 192 MPa.

It is evident from Equation 10 that the risk of rupture is strongly dependent on the load factor k. The load factor can be considered a measure of the uniformity of the stress distribution and is defined from Equations 9 and 10 as

$$k = \int_V \left(\frac{\sigma}{\sigma_{max}}\right)^m \frac{dV}{V} \quad (12)$$

The load factor has been tabulated [7] for rectangular cross-section specimens subjected to various loading conditions. For uniaxial tension, $k = 1$. For a beam of rectangular cross-section, length l, width b taken as unity and height $2h$, subjected to a uniform load, the load factor can be evaluated from Equation 12 as follows:

$$k = \frac{1}{V}\int_0^l \int_0^h \left(\frac{y}{h}\right)^m dy\, dx \quad (13)$$

where the coordinates x and y correspond to the beam length and depth, respectively. The stress σ in Equation 12 is given from elementary beam theory as $\sigma = \sigma_{max} y/h$. The solution to Equation 13 is

$$k = \frac{l}{V}\left(\frac{1}{h}\right)^m \left(\frac{1}{m+1}\right) h^{m+1} \quad (14)$$

Taking $V = 2lh$, the load factor becomes

$$k = \frac{1}{2(m+1)} \quad (15)$$

When beams of a non-rectangular cross-section, such as cylindrical beams, are subjected to uniform bending, the integration of Equation 12 becomes more difficult because of the non-uniformity of the stress through the depth of the beam. For cylindrical beams of radius r, integrals of the form $\int_0^r y^m (r^2 - y^2)^{1/2}\, dy$ must be evaluated. Such integrals can be solved for m equal to an integer from the following expression [8]:

$$\int_0^r y^m (r^2 - y^2)^{n/2}\, dy = \tfrac{1}{2} r^{n+m+1} \Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+2}{2}\right)\Big/\Gamma\left(\frac{m+n+3}{2}\right) \quad (16)$$

where $\Gamma(n)$ is the gamma function. In terms of this integral using $n = 1$, the load factor k for a cylindrical beam of radius r and length l subjected to uniform bending is given by

$$k = \frac{1}{2\pi^{1/2}} \Gamma\left(\frac{m+1}{2}\right)\Big/\Gamma\left(\frac{m+4}{2}\right) \quad (17)$$

The evaluation of Equation 17 for various values of m is shown in Table III.

TABLE III Load factors for a cylindrical beam

m	k
5	4.85×10^{-2}
10	2.05×10^{-2}
15	1.19×10^{-2}
20	8.01×10^{-3}
22	7.01×10^{-3}
25	5.85×10^{-3}

The Weibull modulus for the SiC fibres evaluated in this investigation was $m = 22.3$ ($m = 22$ being the nearest integer), corresponding to a load factor $k = 7.01 \times 10^{-3}$ in uniform bending and, of course, $k = 1$ for uniform tension. For equal total volumes under load, the tension and pure bending strength should be related by $(1/k)^{1/m}$. Because the tested lengths in tension and bending were somewhat different (12.7 and 7.6 cm, respectively), the predicted bend strength from known uniaxial tension strengths is given by

$$\sigma_{bending} = \sigma_{tension}\left(\frac{V_{tension}}{V_{bending}}\frac{1}{k}\right)^{1/m} \quad (18)$$

Using $k = 7.01 \times 10^{-3}$, $m = 22$ and $V_{tension}/V_{bending} = 1.67$ the predicted bending strength is

$$\sigma_{bending} = 1.28\sigma_{tension} \quad (19)$$

From above, the experimental value of the uniaxial tensile strength of the SiC filaments was 3250 ± 146 MPa, and the measured flexure strength 4320 ± 192 MPa. From Equation 19, the equivalent predicted bending strength is 4160 MPa, which is 3.8% lower than observed. This is considered excellent agreement.

It may be noted that, rather than considering the material flaws implicit in the Weibull weakest-link model as uniformly distributed throughout the volume of the filament, one can assume a surface distribution of flaws. In this case, the risk of rupture (Equation 9) becomes

$$R = \int_A \left(\frac{\sigma}{\sigma_0}\right)^m dA \quad (20)$$

Using the same procedure as before, one can find a load factor based on this relationship, and it is found that k_A for $m = 22$ is 0.084. The computed bending strength, 3710 MPa, underestimates the observed filament bend strength by 16%, leading to the tentative conclusion that for this filament population the material flaws are randomly distributed throughout the filament volume.

One final point may be made regarding the comparison of uniaxial tensile and flexural data. The Weibull modulus, m, was determined by a probability plot of the ordering strength data; the slope of this line by a least-squares fitting techniques was found to yield $m = 22.3$. Another less precise estimate of the Weibull modulus is given [6] by

$$m = \frac{\sigma_m}{\text{S.D.}} \tag{21}$$

where σ_m is the mean strength and S.D. the standard deviation. Using the values of the mean bending strengths and their standard deviation, a value of $m = 22.5$ is obtained. This is in excellent agreement with the value obtained from the ordered tensile data.

4. Conclusion

It may be concluded from the results of this investigation that multiple pure bending tests of tapes of high-strength, high-modulus filaments yield strength values that differ by only about 3.8% from uniaxial tensile data. In addition, the standard deviation of such bending tests, and hence the Weibull modulus, is similar to that obtained in uniaxial tensile tests.

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